Transformation of algebraic expressions

How to factor the polynomials?

PROCESS:

I) ax + ay +az = a(x+y+z) and xa+ya+za=(x+y+z)a law of distributivity

II) Formulas:

$$A^2 - B^2 = (A - B) \cdot (A + B)$$

$$A^2 \pm 2AB + B^2 = (A \pm B)^2$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$A^3 + 3A^2B + 3AB^2 + B^3 = (A^3 + B^3)$$

$$A^3 - 3A^2B + 3AB^2 - B^3 = (A^3 - B^3)$$

$$A^3 \pm 3A^2B + 3AB^2 \pm B^3 = (A \pm B)^3$$

III) Joining “two with two”, “three with three” …

EXAMPLES:

I) Law of distributivity

Factor the polynomials:

1) $5a + 5b = 5(a + b)$

2) $2a + 4b = 2(a + 2b)$

3) $a^2 - a = a(a - 1)$ **Watch out: When you see that nothing remains, write 1!**

4) $14ab^3 - 7a^2b = 7ab(2b^2 - a)$ How we do this?

$$\boxed{7} \cdot 2 \cdot \boxed{a} \cdot \boxed{b} \cdot b \cdot b \qquad \boxed{7} \cdot \boxed{a} \cdot a \cdot \boxed{b}$$

$14ab^3 = \underline{7} \cdot 2 \cdot \underline{a} \cdot \underline{b} \cdot b \cdot b$ and

$7a^2b = \underline{7} \cdot \underline{a} \cdot a \cdot \underline{b}$

Underline the same and pull them before bracket!

5)

$3x^2y + 6xy^2 - 3xy =$

$\underline{3} \cdot \underline{x} \cdot x \cdot \underline{y} + \underline{3} \cdot 2 \cdot \underline{x} \cdot \underline{y} \cdot y - \underline{3} \cdot \underline{x} \cdot \underline{y} = 3xy(x + 2y - 1)$

6)

$18a^3b^2 - 15a^2b^3 + 9a^3b^3 =$

$6 \cdot \underline{3} \cdot \underline{a} \cdot \underline{a} \cdot a \cdot \underline{b} \cdot \underline{b} - 5 \cdot \underline{3} \cdot \underline{a} \cdot \underline{a} \cdot \underline{b} \cdot \underline{b} \cdot b + 3 \cdot \underline{3} \cdot \underline{a} \cdot \underline{a} \cdot a \cdot \underline{b} \cdot \underline{b} \cdot b =$

$= 3a^2b^2(6a - 5b + 3ab)$

Of course, we can do our thinking as:

From 18, 15 and 9 we can take 3
From a^3, a^2 and a^3 we can take a^2
From b^2, b^3 and b^3 we can take b^2

Together $\boxed{3a^2b^2}$.

$18a^3b^2 - 15a^2b^3 + 9a^3b^3 = 3a^2b^2(6a - 5b + 3ab)$

7) $a^x + a^{x+1} = a^x + a^x \cdot a^1 = a^x(1+a)$

8) $a^{m+1} - a = a^m \cdot a^1 - a = a(a^m - 1)$

9) $4x^{a+2} + 12x^a = 4x^a \cdot x^2 + 12x^a$
$= 4x^a(x^2 + 3)$

10) $12x^{2n+3} + 16x^{n+1} = 12x^{2n} \cdot x^3 + 16x^n \cdot x^1$
$= 4x^n \cdot x(3x^n \cdot x^2 + 4)$
$= 4x^{n+1}(3x^{n+2} + 4)$

II)

Factor the polynomials:

$$\boxed{A^2 - B^2 = (A-B)\cdot(A+B)}$$

1) $x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$

2) $9 - a^2 = 3^2 - a^2 = (3-a)(3+a)$

3) $x^2 - 1 = x^2 - 1^2 = (x-1)(x+1)$

4) $y^2 - 144 = y^2 - 12^2 = (y-12)(y+12)$

5) $4x^2 - 9 = 2^2x^2 - 3 = (2x)^2 - 3^2 = (2x-3)(2x+3)$

6) $25x^2 - 16y^2 = 5^2x^2 - 4^2y^2 = (5x)^2 - (4y)^2 = (5x-4y)(5x+4y)$

7) $\frac{1}{16}x^2 - \frac{9}{25}y^2 = \frac{1^2}{4^2}x^2 - \frac{3^2}{4^2}x^2 - \frac{3^2}{5^2}y^2 = \left(\frac{1}{4}x - \frac{3}{5}y\right)\left(\frac{1}{4}x + \frac{3}{5}y\right)$

8) $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 - y^2)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2)$

$\boxed{x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)}$ MEMORIZE!

9) $16a^4 - 1 = 2^4 a^4 - 1^4$

$= (2a)^4 - 1^4$, WE CAN USE $x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$;

$2a = x$ AND $1 = y$

$= (2a - 1)(2a + 1)((2a)^2 + 1^2)$

$= (2a - 1)(2a + 1)(4a^2 + 1)$

$\boxed{A^2 + 2AB + B^2 = (A + B)^2}$ **AND** $\boxed{A^2 - 2AB + B^2 = (A - B)^2}$

1) $x^2 + 8x + 16 = ?$

$A^2 = x^2 \Rightarrow A = x$

$B^2 = 16 \Rightarrow B = 4$

$2 \cdot AB = 2 \cdot x \cdot 4 = 8x$

$x^2 + 8x + 16 = (x + 4)^2$

2) $x^2 - 10x + 25 = ?$

$A^2 = x^2 \Rightarrow A = x$

$B^2 = 25 \Rightarrow B = 5$

$2AB = 2 \cdot x \cdot 5 = 10x$

$x^2 - 10x + 25 = (x - 5)^2$

3) $64 + 16y + y^2 = (8 + y)^2$

4) $a^2 + 4ab + 4b^2 = (a + 2b)^2$

5) $a^2 - 6ab + 9b^2 = (a - 3b)^2$

6) $4x^2 - 20xy + 25y^2 = (2x - 5y)^2$

7) $0,25 - 0,1a + 0,01a^2 = (0,5 - 0,1a)^2$ jer je

$A^2 = 0,25 \Rightarrow A = 0,5$

$B^2 = 0,01a^2 \Rightarrow B = 0,1a$

8) $0,04a^2 + 0,8ab + 4b^2 = (0,2a + 2b)^2$

$$\boxed{A^3 - B^3 = (A - B)\cdot(A^2 + AB + B^2)}$$

1) $x^3 - 8 = ?$

$x^3 - 8 = x^3 - 2^3$

$x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + x\cdot 2 + 2^2) = (x-2)(x^2 + 2x + 4)$

2) $x^3 - 216 = x^3 - 6^3 = (x-6)(x^2 + x\cdot 6 + 6^2) = (x-6)(x^2 + 6x + 36)$

3) $64 - y^3 = 4^3 - y^3 = (4-y)(4^2 + 4y + y^2) = (4-y)(16 + 4y + y^2)$

4) $125x^3 - 1 = 5^3x^3 - 1^3 = (5x)^3 - 1^3 =$

$= (5x-1)\left((5x)^2 + 5x\cdot 1 + 1^2\right)$

$= (5x-1)(25x^2 + 5x + 1)$

5) $(a+3)^3 - 8 = (a+3)^3 - 2^3 =$ pay attention: $\boxed{a+3 = A}$, $\boxed{2 = B}$

$= (a+3-2)\left((a+3)^2 + (a+3)\cdot 2 + 2^2\right)$

$= (a+1)(a^2 + 6a + 9 + 2a + 6 + 4)$

$= (a+1)(a^2 + 8a + 19)$

$$\boxed{A^3 + B^3 = (A + B)(A^2 - AB + B^2)}$$

1) $x^3 + 343 = x^3 + 7^3 = (x+7)(x^2 - x\cdot 7 + 7^2) = (x+7)(x^2 - 7x + 49)$

2) $64a^3 + 1 = (4a)^3 + 1^3 = (4a+1)\left((4a)^2 - 4a\cdot 1 + 1^2\right) = (4a+1)(16a^2 - 4a + 1)$

3) $27x^3 + y^3 = (3x)^3 + y^3 = (3x+y)\left((3x)^2 - 3x\cdot y + y^2\right) = (3x+y)(9x^2 - 3xy + y^2)$

4) $\underbrace{(x+1)}_{A}{}^3 + \underbrace{(y-2)}_{B}{}^3 = (x+1+y-2)\cdot\left[(x+1)^2 - (x+1)(y-2) + (y-2)^2\right]$

$= (x+y-1)\left[x^2 + 2x + 1 - (xy - 2x + y - 2) + y^2 - 4y + 4\right]$

$= (x+y-1)\left[x^2 + 2x + 1 - xy + 2x - y + 2 + y^2 - 4y + 4\right]$

$= (x+y-1)\left[x^2 + 4x + y^2 - 5y - xy + 7\right]$

5) $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)\left((x^2)^2 - x^2y^2 + (y^2)^2\right) = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$\boxed{A^3 \pm 3A^2B + 3AB^2 \pm B^3 = (A \pm B)^3}$$

1) $\underbrace{8x^3}_{\substack{\uparrow \\ A^3}} + \underbrace{12x^2y}_{\substack{\uparrow \\ 3A^2B}} + \underbrace{6xy^2}_{\substack{\uparrow \\ 3AB^2}} + \underbrace{y^3}_{\substack{\uparrow \\ B^3}} =$ [$A^3 = 8x^3$ then $A = 2x$ and $B^3 = y^3$ then $B = y$]

$= (2x + y)^3$

2) $x^3 - 12x^2y + 4xy^2 - 64y = (x - 4y)^3$ because

$A^3 = x^3 \Rightarrow A = x$

$64y^3 = B^3 \Rightarrow B = 4y$

3)

$125a^3 + 150a^2b + 60ab^2 + 8b^3 = (5a + 2b)^3$ because

$125a^3 = (5a)^3$ and $8b^3 = (2a)^3$

III)

Joining "two with two", "three with three" …

1) $2x + 2y + ax + ay =$

$2(x + y) + a(x + y) = (x + y)(2 + a)$

2) $\underbrace{6ax - 9bx} + \underbrace{8ay - 12by} =$

$3x(2a - 3b) + 4y(2a - 3b) = (2a - 3b)(3x - 3b)(3x + 4y)$

3) $\underbrace{4a^2 + 4a} - ab - b =$ **pay attention on sign**

$4a(a + 1) - b(a + 1) = (a + 1)(4a - b)$

4) $\underbrace{12ab + 20a} - \underbrace{3b - 5} =$

$4a(3b + 5) - 1(3b + 5) = (3b + 5)(4a - 1)$

5) $\underbrace{xa - xb} + \underbrace{yb - ya} =$

$= x(a - b) + y(b - a) =$ [attention, because b-a = - (a-b)]

$= x(a - b) - y(a - b) = (a - b)(x - y)$

6) $2ax+b-2bx-a = a(2x-1)+b(1-2x) = a(2x-1)-b(2x-1) = (2x-1)(a-b)$

7) $8x^2y-2by+2bx-8xy^2 = 8xy\underline{\underline{(x-y)}}+2b\underline{\underline{(x-y)}} = (x-y)(8xy+2b)$

8) $x^2-6x-7=?$

First method of finding a solution: [idea: - 6x = -7x+1x]

$x^2-6x-7 = x^2-7x+1x-7 = x(x-7)+1(x-7) = (x-7)(x+1)$

Second method of finding a solution:

$$\begin{aligned} x^2-6x-7 &= \underbrace{x^2-6x}+3^2-3^2-7 = \\ &= \underbrace{x^2-6x+9}-9-7 \\ &= (x-3)^2-16 \\ &= (x-3)^2-4^2 \\ &= (x-3-4)(x-3+4) \\ &= (x-7)(x+1) \end{aligned}$$

9) $x^2+5x+6=?$

First method of finding a solution: [idea: 5x = 3x + 2x]

$\underbrace{x^2+3x}+\underbrace{2x+6} = x(x+3)+2(x+3) = (x+3)(x+2)$

Second method of finding a solution:

$$\begin{aligned} x^2+5x+6 &= \underbrace{x^2+5x+\left(\frac{5}{2}\right)^2}-\left(\frac{5}{2}\right)^2+6 \\ &= \left(x+\frac{5}{2}\right)^2-\frac{25}{4}+\frac{24}{4} \\ &= \left(x+\frac{5}{2}\right)^2-\frac{1}{4} \\ &= \left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2 \\ &= \left(x+\frac{5}{2}-\frac{1}{2}\right)\left(x+\frac{5}{2}+\frac{1}{2}\right) \\ &= (x+2)(x+3) \end{aligned}$$

10) $x^2 + 7x + 10 = ?$

i) $\underbrace{x^2 + 5x} + \underbrace{2x + 10} = x(x+5) + 2(x+5) = (x+5)(x+2)$

ii) $x^2 + 7x + 10 = x^2 + 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 10$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + \frac{40}{4}$$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{9}{4}$$

$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= \left(x + \frac{7}{2} - \frac{3}{2}\right)\left(x + \frac{7}{2} + \frac{3}{2}\right)$$

$$= (x+2)(x+5)$$

Annotation: quadratic equation $ax^2 + bx + c = 0 \Leftrightarrow a(x - x_1)(x - x_2) = 0$ $\quad x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

give as fast decision! [see quadratic equation]